

1. A car manufacturer has included some improvements in certain model. The number of kms (in thousands) run by the old model without breaking down followed a $N(250;30)$. The car manufacturer wants to verify if the advances have been effective. Aimed at that, it has randomly selected 100 cars of the new model, observing that the average number of kms run by them without problems is 275. Assuming that neither σ nor the probability distribution have changed, test the hypothesis that the improvements realized in the model have been useless or, even, make worse against the situation in which they have been effective. Use a 1% significance level.
2. Certain firm assures that the proportion of its employees having a healthy lifestyle is lower than 20%. To show this it has obtained a s.r.s. of 150 employees asking them about their habits, 21 of them falling in the “yes” section.
 - a. Propose a hypothesis test at a 5% significance level under the point of view of the firm.
 - b. Propose a hypothesis test at a 1% significance level under the point of view of the firm.
 - c. Discuss the results obtained.
 - d. Compute the p-value.
3. The number of litres of drink (in thousands) filled daily by a bottling company is a random variable $N(150;5)$. However, in the last three months some changes have been tried into the filling system. Those changes will be definitely incorporated if there are objective proofs verifying that the volume of drink bottled is higher under the new technique. Aimed at that the production manager has selected a s.r.s. of 25 days, the average number of liters bottled being 153. Assuming no changes in the variance nor in the probability distribution, formulate the corresponding test at $\alpha = 2\%$. Apart from that, compute the p-value.
4. The amount of calcium in milligrammes per litre contained in a bottle of mineral water of 1,5 litres of certain brand follows a $N(4;\sigma)$, according to its label. In order to check such information a clinical laboratory has obtained a s.r.s. with 10 bottles, the volume of calcium included being:

3,9 3,8 4,1 4,2 3,9 4,1 4,2 4,3 4 3,9

In the light of these data, test the null hypothesis that the amount of calcium is consistent with the measure informed in the label at a significance level of 1%. Solve for these cases, computing the p-value at the same time:

 - a. Propose a two-sided test
 - b. Propose a one-sided test to the left
 - c. Propose a one-sided test to the right.
 - d. Compare the results obtained.

5. A canning company sells tuna fish in olive oil with a net weight of 250 gramms in each can, according to its label. Assuming that the variable follows a normal distribution, a delegation of the ministry of consumer affairs has randomly collected 20 tins providing these data:

$$\sum_{i=1}^n x_i = 4946 \qquad \sum_{i=1}^n x_i^2 = 1223638$$

Formulate a decision rule to test the information in the label at $\alpha = 2\%$ and consider these situations:

- A two tailed hypothesis test
 - A one tailed hypothesis test to the left
 - Calculate the p-value in both cases
 - Discuss the results achieved
6. An expert in economic matters believes that the proportion of Spanish families struggling for covering expenses at the end of every month is higher than 70%. In order to prove that he has obtained a s.r.s. with 200 families, 150 of them being in such situation.
- Propose the hypothesis testing the expert would carry out in order to test his conjecture, at $\alpha = 5\%$
 - Compute the p-value
7. The product manager of the canning company in the exercise number 5 wants to prove that the tuna tins verify the company standard which establish that the variance of the tin weight has to be lower than 25. The fact that this standard were not fulfilled might have negative consequences for the company.
- Formulate the appropriate test at a 2% level of significance.
 - Compute the p-value.
8. The daily demand of certain product follows a $N(\mu; \sigma)$. A manager intends to test the null hypothesis that the variance is equal to 50 using a two tailed test at a significance level of 10%. Aimed at that a s.r.s. with 100 observations has been selected, observing this behaviour:

$$\sum_{i=1}^n (x_i - \bar{x})^2 = 4851$$

Do the corresponding test, apart from calculating the p-value.

9. With the objective of checking if there are differences between the grades in Statistics and Mathematics in Business Graduate students, two matched (dependent) samples from 30 students have been obtained. Assuming grades are normally distributed in both subjects, answer:
- Test that the average grade in both subjects is the same at $\alpha = 0,01$
 - Compute the p-value

The grades collected are:

| Student | X: Grade in Statistics | Y: Grade in Accounting |
|---------|------------------------|------------------------|
| 1 | 7 | 6 |
| 2 | 6 | 8 |
| 3 | 4 | 5 |
| 4 | 8 | 3 |
| 5 | 3 | 3 |
| 6 | 2 | 6 |
| 7 | 1 | 5 |
| 8 | 5 | 8 |
| 9 | 6 | 2 |
| 10 | 7 | 3 |
| 11 | 7 | 2 |
| 12 | 4 | 1 |
| 13 | 2 | 4 |
| 14 | 3 | 5 |
| 15 | 6 | 6 |
| 16 | 5 | 5 |
| 17 | 4 | 7 |
| 18 | 7 | 8 |
| 19 | 5 | 8 |
| 20 | 4 | 9 |
| 21 | 3 | 2 |
| 22 | 2 | 10 |
| 23 | 4 | 6 |
| 24 | 6 | 5 |
| 25 | 8 | 6 |
| 26 | 7 | 9 |
| 27 | 2 | 9 |
| 28 | 6 | 8 |
| 29 | 9 | 4 |
| 30 | 8 | 2 |

10. A milk producer wants to show that the market share it enjoys in Asturias is higher than that in Galicia, the difference being beyond 5 percentage points. With that intention it obtains a s.r.s. with 200 people in Asturias and another sample with 150 people in Galicia. The number of respondents saying they drink that brand of milk being 70 in Asturias and 30 in Galicia.
- Do the corresponding test at $\alpha = 0,05$
 - Repeat previous question but now at $\alpha = 0,01$
 - Compute the p-value.
 - Discuss.

11. The wind energy power installed in Castilla La Mancha (X) reaches 2,008 gw/h (gigawatts/hour), meanwhile that in Castilla León (Y) is 1,69 gw/h. According to this the estimated difference in the amount of electric energy produced between both regions should be on average around 0,318 gw/h (source: <http://www.infoeolica.com>). Aimed at testing the hypothesis that the difference is exactly that and assuming that production follows a normal distribution, two independent s.r.s. have been obtained along two days (48 data for each region). Data are:

$$\sum_1^n x_i = 81,12 \quad \sum_1^n x_i^2 = 139,1048 \quad \sum_1^m y_i = 71,72 \quad \sum_1^m y_i^2 = 109,302$$

Answer:

- Formulate the corresponding test at a 5% significance level. Compute the p-value.
 - Elaborate a one tailed test to the left at the same alfa, apart from calculating the p-value.
 - Discuss.
12. In the previous exercise test the equality in the respective variances at $\alpha = 1\%$ following these instructions:
- Apply a two sided test. Compute the p-value.
 - Apply a one sided test to the left and compute the p-value.
 - Discuss.
 - Redo exercise 11.a taking into account the result obtained in 12.a.
13. Redo exercise number 6 and calculate the power of the test.
14. Redo exercise number 7 and calculate the power of the test.
15. The owner of a fish post in a market is certain he sells daily more than 100 kgs of small hake on average. To verify that he tells a friend that he has randomly selected 50 days, having sold 101 kgs of small hake on average with a standard deviation of 30 kgs.
- Propose the corresponding test at a 2% significance level. At the same time, compute the p-value.
 - Discuss how a reduction in the sampling standard deviation would affect the previous test. Might that situation change the final decision?